2.14 Sine and Cosine Integrals

A. Purpose

Given x, these subprograms compute the Sine Integral, $\mathrm{Si}(x)$, the Cosine Integral, $\mathrm{Ci}(x)$, and an entire function, $\mathrm{Cin}(x)$ related to the cosine integral. These functions are defined as

$$\mathrm{Si}(x) = \int_0^x \frac{\sin(s)}{s}\, ds,$$

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos(s)}{s}\, ds, \quad \text{and}$$

$$\mathrm{Cin}(x) = \int_0^x \frac{1-\cos(s)}{s}\, ds,$$

$\mathrm{Ci}(x)$ and $\mathrm{Cin}(x)$ satisfy the relation $\mathrm{Ci}(x) + \mathrm{Cin}(x) = \ln x + \gamma$, where γ is Euler's constant, approximately 0.57721 56649. To calculate the related sine integral defined by $\mathrm{si}(x) = -\int_x^\infty \frac{\sin(s)}{s}\, ds$ use the identity $\mathrm{si}(x) = \mathrm{Si}(x) - \pi/2$.

Reference [1] provides further discussion of the properties of the Sine and Cosine integrals.

B. Usage

B.1 Program Prototype, Single Precision

REAL X, SSI, SCI, SCIN, T

Assign a value to X and use one of the following function references.

To compute the Sine Integral $\mathrm{Si}(x)$:

T = SSI(X)

To compute the Cosine Integral $\mathrm{Ci}(x)$ for $x > 0$:

T = SCI(X)

To compute the entire function $\mathrm{Cin}(x)$ related to the Cosine Integral:

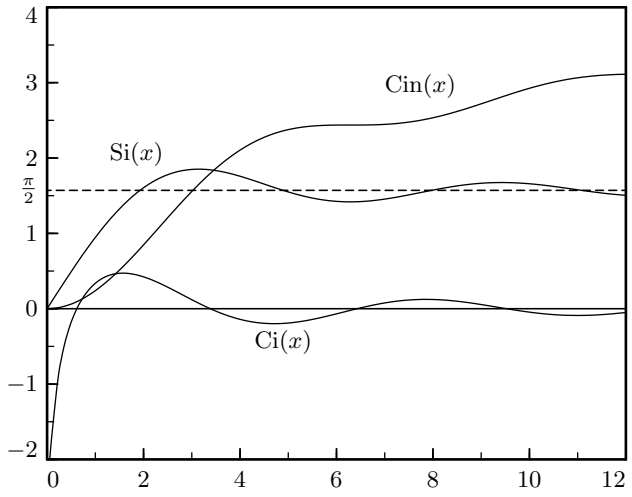
T = SCIN(X)

B.2 Argument Definitions

X [in] Argument of function.

B.3 Modification for Double Precision

For double precision usage change the REAL type statement to DOUBLE PRECISION and change the function names to DSI, DCI or DCIN respectively.



C. Examples and Remarks

See DRSSI and ODSSI for an example of the usage of these subprograms.

$\mathrm{Si}(x)$ is an odd function and $\mathrm{Cin}(x)$ is an even function. $\mathrm{Ci}(x)$ is not defined for $x \le 0$.

The notations for these functions vary. We use the notation of [1].

D. Functional Description

The computer approximations for these functions use Chebyshev polynomial expansions.

These subprograms were tested on the IBM PC/AT by comparing the results to tables in [1].

References

1. Milton Abramowitz and Irene A. Stegun, **Handbook of Mathematical Functions**, *Applied Mathematics Series 55*, National Bureau of Standards (1966) Chapter 5, 231–244.

E. Error Procedures and Restrictions

The subprograms SSI and SCIN detect no error conditions. The subprogram SCI issues an error message if $x \le 0$. The error message is issued by way of the error message processor at level 0, and the returned function value is 0.0. See Chapter 19.2 for further description of the error message processor.

F. Supporting Information

The source language is ANSI Fortran 77.

Subprograms designed and developed by E. W. Ng, JPL, 1970. Modified by W. V. Snyder, JPL, 1989.

Entry	Required Files
DCI	DCI, DCPVAL, DERM1, DERV1, ERFIN, ERMSG
DCIN	DCI, DCPVAL, DERM1, DERV1, ERFIN, ERMSG
DSI	DCPVAL, DSI
SCI	ERFIN, ERMSG, SCI, SCPVAL, SERM1, SERV1
SCIN	ERFIN, ERMSG, SCI, SCPVAL, SERM1, SERV1
SSI	SCPVAL, SSI

DRSSI

```
      program DRSSI
c>> 1994-10-19 DRSSI   Krogh  Changes to use M77CON
c>> 1991-11-19  DRSSI   CLL
c>> 1989-04-26  DRSSI   Snyder at JPL, Original Code
c--S replaces "?":  DR?SI, ?CI, ?CIN, ?SI
c
c     Demonstration driver for Sine and Cosine integrals.
c
      real              SCI, SCIN, SSI, X
      integer I
      external SCI, SCIN, SSI
c
c     Print values of Si(x), Ci(x) and Cin(x)
c     for X = 1.0 (1.0) 15.0
c
      print '('' X'',11x,''Si(x)'',11x,''Ci(x)'',11x,''Cin(x)''/)'
      do 10 i = 1, 15
         x = real(i)
         print '(f4.0,3f16.7)', x, ssi(x), sci(x), scin(x)
   10 continue
      stop
      end
```

ODSSI

X	Si(x)	Ci(x)	Cin(x)
1.	0.9460829	0.3374040	0.2398117
2.	1.6054132	0.4229809	0.8473819
3.	1.8486525	0.1196299	1.5561981
4.	1.7582031	−0.1409814	2.1044915
5.	1.5499313	−0.1900294	2.3766830
6.	1.4246874	−0.0680568	2.4370320
7.	1.4545966	0.0766957	2.4464302
8.	1.5741869	0.1224341	2.5342231
9.	1.6650400	0.0553482	2.7190921
10.	1.6583476	−0.0454559	2.9252567
11.	1.5783069	−0.0895629	3.0646739
12.	1.5049714	−0.0497799	3.1119022
13.	1.4993616	0.0267637	3.1154013

14.	1.5562110	0.0693953	3.1468778
15.	1.6181943	0.0462780	3.2389879